Fluctuating Lift and Moment Coefficients for Cascaded Airfoils in a Nonuniform Compressible Flow

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The effects of compressibility on both the fluctuating lift and the fluctuating moment coefficients for cascaded airfoils due to an upstream nonuniformity are determined by obtaining a solution to the time-dependent, compressible, two-dimensional partial differential equation which describes the perturbation velocity potential. This is accomplished through an application of Fourier-transform theory, with the resulting integral solution equation evaluated numerically by a matrix-inversion technique. The results presented show the variation in both the fluctuating lift and the fluctuating moment coefficients over the mean cascade inlet Mach number range of 0.0 (incompressible) to 0.9 with the cascade solidity, cascade stagger angle, interblade phase angle and reduced frequency as parameters.

Nomenclature

a_∞ = mean cascade inlet speed of sound
j = $(-1)^{1/2}$
k = $\omega C/2U_\infty$
m,n = integers
m_b = mass per unit length of the airfoil
s = Fourier parameter
t = time
v = arbitrary small quantity
$w(x)$ = axial dependence of the upwash velocity
x, z = dimensionless rectangular coordinates, $x = X/C$, $z = Z/C$
C = airfoil chord
C_L = fluctuating lift coefficient defined by Eq. (14)
C_M = fluctuating moment coefficient defined by Eq. (15)
D = distance between leading edges of adjacent blades as measured in the x direction
E = modulus of elasticity
F_i = generalized force of the ith mode
I = structural moment of inertia
K = kernel function
L = span of airfoil
M_i = generalized mass of the ith mode
M_∞ = mean cascade inlet Mach number
S = cascade spacing
U_∞ = freestream velocity
X,Z = dimensional rectangular coordinates
Y = dimensional airfoil span
$W(X,t)$ = upwash on airfoil
$\zeta_i(Y)$ = normalized natural mode shapes of airfoil
$\eta_i(t)$ = normal airfoil coordinates
ξ = $\xi^2 = -(1-M_\infty{}^2)s^2 + 4kM_\infty{}^2 s + 4k^2M_\infty{}^2$
σ = interblade phase angle
$\phi(X,Z)$ = two-dimensional perturbation velocity potential
ω = forced angular frequency
$Z_b(Y,t)$ = forced displacement of airfoil
Ω_i = natural angular frequency of airfoil
$\Delta p(x)$ = nondimensional axial pressure difference across airfoil chordline
ρ_∞ = mean cascade inlet density
β = cascade stagger angle
$\Phi(x,z,t)$ = unsteady perturbation velocity potential
$Im(L,M)$ = imaginary part of complex quantity L,M
$Re(L,M)$ = real part of complex quantity L,M

Superscript

$(\ )^*$ = Fourier transform

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Introduction

THE propagation of upstream unsteady aerodynamic disturbances past a compressor or fan blade row is an important consideration in the design of aircraft engines. This problem has significance with regard to an unsteady or distorted flowfield entering an axial-flow compressor as well as to the downstream effects caused by the unsteady motion of an upstream blade row, e.g., the aerodynamic interference between a rotor and stator.

Kemp and Sears[1,2] described the aerodynamic interference between a stator and rotor with the simplifications of two-dimensional, incompressible isolated-airfoil theory. The unsteady effects due to the relative motion of the steady-state parts of the flowfield around the rotor and stator were considered, but the mutual interaction of the circulation around the cascaded airfoils was neglected. Mani[3] extended the analyses of Kemp and Sears by allowing for cascade and compressibility effects.

The description of the unsteady flow due to a cascade of airfoils oscillating about some point in an otherwise uniform flowfield has been considered by flutter specialists.[4-6] This condition, while closely related, is not the same as the problem being treated herein, i.e., the flow through a cascade of rigid airfoils with upstream unsteady disturbances carried downstream with the flow. Whitehead[6] and Schorr and Reddy[7] carried out an incompressible potential-flow formulation of this problem. These incompressible analyses are severely limited as applied to present and future high tip-speed fans and compressors wherein compressibility effects are known to be significant. Consequently, this paper presents a compressible formulation of the aforementioned flow problem—upstream unsteady disturbances transported downstream through a cascade of rigid airfoils by a compressible fluid.

Mathematical Model

The mathematical model is concerned with the two-dimensional, unsteady flow of a compressible fluid past a rectilinear cascade. The basic assumptions are 1) the fluid is a perfect gas; 2) the flowfield is irrotational; 3) the thin-airfoil approximations are appropriate.

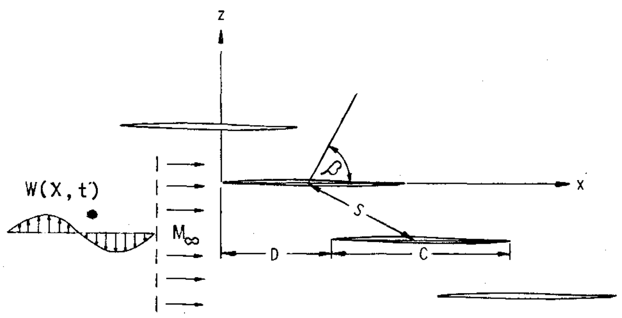

Fig. 1 Two-dimensional cascade and flowfield.

Since this paper is concerned with the effects of small unsteady disturbances convected downstream past a cascade of thin airfoils by a compressible fluid flow, it is assumed that there is a basic uniform flow past the airfoil cascade with unsteady normal velocity fluctuations superimposed. The source of the unsteady fluctuations is located upstream and the oscillations in the velocity normal to the airfoil surfaces are convected downstream with the uniform compressible flow. See Fig. 1.

The thin-airfoil assumptions taken together with the approximation of small unsteady, compressible perturbations on the basic uniform two dimensional compressible flow lead to the following partial differential equation for the unsteady perturbation velocity potential function:

$$(1-M_\infty^2)\frac{\partial^2\Phi}{\partial X^2}+\frac{\partial^2\Phi}{\partial Z^2}-\frac{2M_\infty}{a_\infty}\frac{\partial^2\Phi}{\partial X\partial t}-\frac{1}{a_\infty^2}\frac{\partial^2\Phi}{\partial t^2}=0 \quad (1)$$

where $\Phi(X,Z,t)$ denotes the unsteady perturbation velocity potential function, M_∞ and a_∞ are the mean cascade inlet Mach number and sound speed, respectively, t denotes the time, and the coordinates X and Z are indicated in Fig. 1. It should be noted that the solution to Eq. (1) with no further simplifications will be obtained by means of complex Fourier-integral transform theory.

The upstream unsteady fluctuations being considered herein are mathematically equivalent to prescribing normal relative velocities (or upwash) of the form indicated in Eq. (2) on each of the airfoils of the cascade. Hence, the appropriate boundary conditions are the following:

$$W(X,t)=v\exp[j\omega(t-X/U_\infty)] \quad (2)$$

on the surfaces of each airfoil with appropriate phase difference as detailed below where $W(X,t)$ is the normal relative velocity (the upwash velocity), $j=(-1)^{1/2}$ and v is an arbitrarily small quantity.

Not only unsteady inlet flows and aerodynamic interference between a rotor and stator, but also steady-state circumferential distortion can be described by the relative normal velocity, $W(X,t)$. A steady-state circumferential distortion can be decomposed by a Fourier series approach into an unsteady or oscillatory flow with some phase lag between the relative normal velocity distributions on each airfoil. A complete description of the oscillatory inlet flow and the steady-state circumferential distortion with regard to the form of the relative normal velocity specified in Eq. (2) is presented in Ref. 7.

To simplify the solution procedure, the approximation is made that the boundary conditions specified in Eq. (2) are satisfied at the mean position of each airfoil, i.e, the position about which each surface moves, as indicated by the X-Z coordinate system of Fig. 1.

The linearity of the differential equation and the form of the boundary conditions, Eqs. (1) and (2), result in the following formulation for the velocity-perturbation potential function:

$$\Phi(X,Z,t)=\phi(X,Z)\exp(j\omega t) \quad (3)$$

Note that Φ has been separated into a product of spatial and time-dependent functions. The substitution of the above velocity potential function into Eqs. (1) and (2) leads to the following differential equation and boundary conditions:

$$(1-M_\infty^2)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}-4jkM_\infty^2\frac{\partial\phi}{\partial x}+4k^2M_\infty^2\phi=0 \quad (4)$$

$$\partial\phi/\partial z=Cw(x)=Cv\exp(-j2kx) \quad (5)$$

on each airfoil surface with appropriate phase difference as detailed below where k denotes the reduced frequency, $k=\omega C/2U_\infty$, C denotes the airfoil chord-length, and x and z are the nondimensional coordinate lengths defined as $x=X/C$ and $z=Z/C$.

Fourier Transform Analysis

The solution to Eqs. (4) and (5) will be obtained through an application of complex Fourier-transform theory. The complex Fourier-transform pair is defined as

$$f^*(s)=\int_{-\infty}^{\infty}f(x)\exp(-jxs)dx \quad (6)$$

and

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}f^*(s)\exp(jxs)ds \quad (7)$$

where s is the Fourier parameter and the superscript * denotes the Fourier transformation of that quantity.

The application of this transformation on the independent variable x, defined in Eq. (6), to the partial differential equation describing the perturbation velocity potential function, Eq. (4), yields the following ordinary differential equation:

$$d^2\phi^*/dz^2+\xi^2\phi^*=0 \quad (8)$$

where $\xi^2=-(1-M_\infty^2)s^2+4kM_\infty^2s+4k^2M_\infty^2$.

The general solution to Eq. (8) is found to be

$$\phi^*(s,z)=A\sin(\xi z)+B\cos(\xi z) \quad (9)$$

where A and B are integration constants which must be determined from the transformation of the boundary conditions.

At this point in the analysis, it is assumed that the interblade phase angle, σ, is constant. This means that the time-dependent perturbations which occur at $(X+D,\ Z+S\cos\beta)$ lead the values of the same perturbations at (X,Z) by the constant interblade phase angle σ. See Fig. 1. This assumption involves no loss of generality since any required motion of the airfoils can be obtained through a superpositioning of a number of solutions of the type being considered. Moreover, the normal relative velocities on the pressure and suction surfaces are assumed to be equal, i.e., the Kutta condition holds.

The solution for $\phi^*(s,z)$ can now be found as a function of the transformation of the axial dependence of the relative normal velocities, $w(x)$

$$\phi^*(s,z)=\frac{Cw^*(s)}{\xi\sin(\xi S/C)}[\cos(\xi(zS/C))-\exp(j\sigma-js D/C)\cos(\xi z)] \quad (10)$$

where $w^*(s)$ denotes the complex Fourier transformation of the axial dependence of the relative normal velocities, as defined in Eq. (5), S is the cascade spacing and D is the distance between the leading edges of adjacent blades as measured in the X direction.

The remaining step in the analysis involves the use of the unsteady Euler equation to relate the relative normal velocities on the airfoil surfaces $w(x)$ to the unsteady perturbation pressure differences across the airfoil chordline, $\Delta p(x)\exp(j\omega t)$, e.g., see Refs. 4 and 7. The application of the complex Fourier transformation results in Eq. (11) which relates the transformed relative normal velocities on the airfoil surfaces, $w^*(s)$, and the transformed axial dependence of the unsteady pressure perturbation, $\Delta p^*(s)$.

$$\frac{w^*(s)}{U_\infty} = \Delta p^*(s)\left[\frac{j\xi \sin(\xi S/C)}{2(2k+s)[\cos(\xi S/C) - \cos(\sigma - sD/C)]}\right] \quad (11)$$

The axial dependence of the unsteady perturbation pressure difference across the airfoil chordline, $\Delta p(x)$, may be determined as the solution of Eqs. (12) and (13) by taking the inverse Fourier transformation defined in Eq. (7).

$$\frac{v}{U_\infty}\exp(-j2kx) = \int_0^1 K(x-s)\Delta p^*(s)ds \quad (12)$$

and

$$K(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{s+2k}\,\frac{j\xi \sin(\xi S)\exp(jxs)}{\cos(\xi S/C) - \cos(\sigma - sD/C)}ds$$

The inverse Fourier integral for the kernel function $K(x)$ can now be evaluated numerically and also written in the following form:

$$K(x) = \frac{-1}{4\pi(1-M_\infty^2)^{1/2}}\left[\frac{1-M_\infty^2}{x} + j2k\ln x + F_1(x)\right] \quad (13)$$

It should be noted that this particular method of writing the kernel function explicitly displays the singular behavior in the axial coordinate in the first two terms. The function $F_1(x)$ is well behaved and contains the effects of the aerodynamic interference of all the blades of the cascade on the reference airfoil, including the combinations of flow and cascade parameters which make $F_1(x)$ infinite independent of the axial coordinate due to a finite speed of sound, as discussed in Refs. 9 and 10. The singularities explicitly displayed in the preceding formulation are the

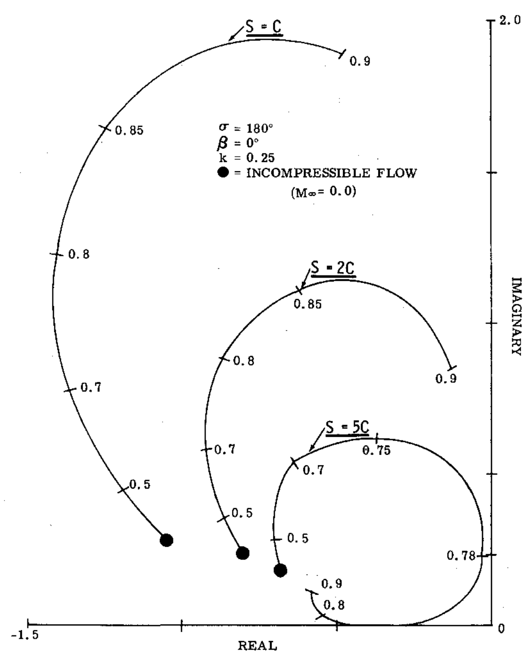

Fig. 2 Solidity effect on the complex fluctuating lift coefficient with the mean cascade inlet Mach number as parameter.

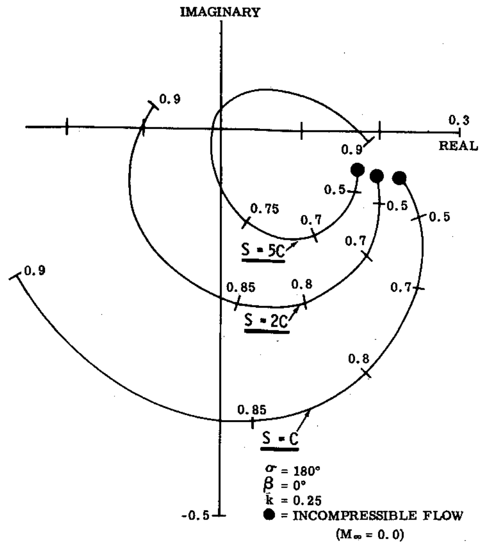

Fig. 3 Solidity effect on the complex fluctuating moment coefficient with the mean cascade inlet Mach number as parameter.

logarithmic and Cauchy-type singularities determined from previous isolated-airfoil solutions. As can be clearly seen from Eq. (13), these singularities are reproduced exactly.[4] It should be noted that since the kernel function is singular, Eq. (12) is also singular at $x = s$. Hence, in Eq. (12), the principal value of the integral is required.

For a given cascade, Eq. (12), a singular integral equation, is numerically solved by evaluating the kernel function defined by Eq. (13), assuming values for the reduced frequency and the interblade phase angle, and then applying a matrix-inversion technique to determine $\Delta p(x)$. The complex fluctuating lift and moment coefficients, C_L and C_M, are then obtained by means of the integrations of $\Delta p(x)$ defined by Eqs. (14) and (15). This solution procedure is analogous to that described in Ref. 6

$$C_L = \int_{-1/2}^{1/2}\Delta p(x)dx/\pi \quad (14)$$

$$C_M = \int_{-1/2}^{1/2} x\Delta p(x)dx/\pi \quad (15)$$

Results

The nondimensional fluctuating lift and moment coefficients defined by Eqs. (14) and (15) have been evaluated. They agree to an acceptable accuracy with the potential-flow results for both isolated airfoils[4] and cascaded airfoils in the incompressible-flow regime.[6]

The effects of cascade solidity, cascade stagger angle, interblade phase angle, and reduced frequency on the complex fluctuating lift and moment coefficients, C_L and C_M, over a mean cascade inlet Mach-number range of 0.0 (incompressible) to 0.9 are indicated in Figs. 2–9. For comparison purposes, the incompressible flow results presented in Ref. 6 are also indicated in these figures. It should be noted that the quantities that are physically meaningful are the absolute magnitudes of the complex fluctuating lift and moment coefficients, $|C_L|$ and $|C_M|$, and their respective phase angles, $\tan^{-1}[Im(C_L)/Re(C_L)]$ and $\tan^{-1}[Im(C_M)/Re(C_M)]$.

Figures 2 and 3 indicate the effect of the solidity of the cascade (airfoil chord/cascade spacing) on the fluctuating lift and moment coefficients, respectively, as the mean cascade inlet Mach number is varied from 0.0 to 0.9 for a reduced frequency equal to 0.25, zero stagger, and a 180° interblade phase angle. It can be seen that as the solidity of the cascade increases, the absolute magnitudes of both the fluctuating lift and the fluctuating moment coeffi-

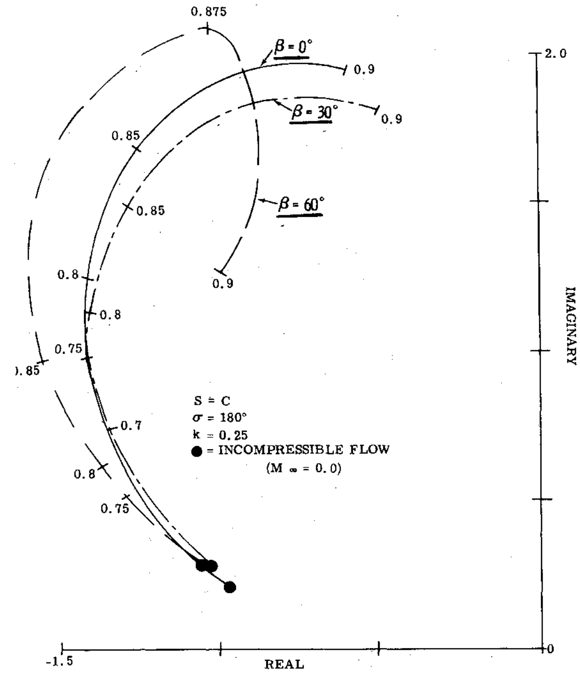

Fig. 4 Effect of stagger angle on the complex fluctuating lift coefficient with the mean cascade inlet Mach number as parameter.

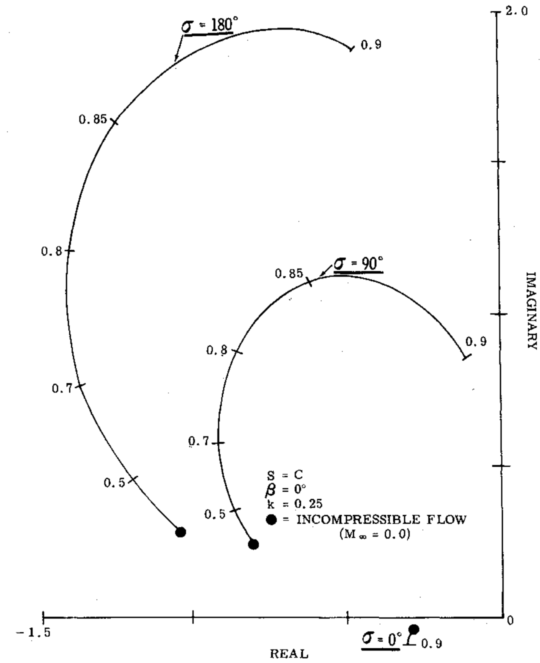

Fig. 6. Interblade-phase-angle effect on the complex fluctuating lift coefficient with the mean cascade inlet Mach number as parameter.

cients increase and attain maximum values at values of the mean cascade inlet Mach number that are less than one for the parameters considered. The mean Mach numbers at which these maximum absolute values of the lift and moment coefficients are attained can also be seen to increase with increasing values of cascade solidity. In addition, there are found relatively large changes in the fluctuating lift and moment coefficients with relatively small changes in the mean Mach number near the values of mean Mach number at which the maximum absolute magnitudes of these coefficients are attained. This indicates that the complex phases of the fluctuating lift and moment coefficients are changing rapidly with the mean cascade inlet Mach number near to the peak-coefficient mean Mach numbers. These rapid phase changes with mean Mach number are analogous to the aerodynamic resonance Mach numbers discussed in Ref. 10.

Figures 4 and 5 show the effect of the cascade stagger angle on the fluctuating lift and moment coefficients, respectively, as the mean cascade inlet Mach number is varied over the compressible subsonic flow regime for a reduced frequency equal to 0.25, a cascade solidity of one, and a 180° interblade phase angle. The absolute magnitudes of the lift and moment coefficients attain their maximum values at mean cascade inlet Mach-numbers that are less than one and, near to these peak mean Mach numbers, rapid changes in the lift and moment coefficients with relatively small variations in the mean Mach number, indicative of rapid changes in the complex phases of these coefficients, are found. The behavior of the $\beta = 60°$ results for mean Mach-number values greater than 0.875 is analogous to the aerodynamic resonance that occurs in the wind tunnel wall interference problem.

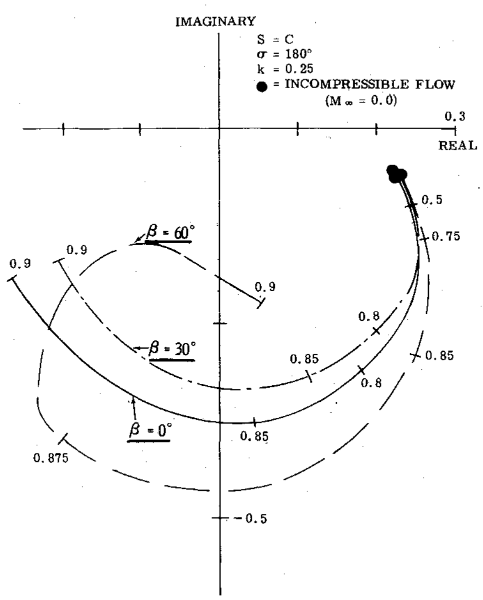

Fig. 5 Effect of stagger angle on the complex fluctuating moment coefficient with the mean cascade inlet Mach number as parameter.

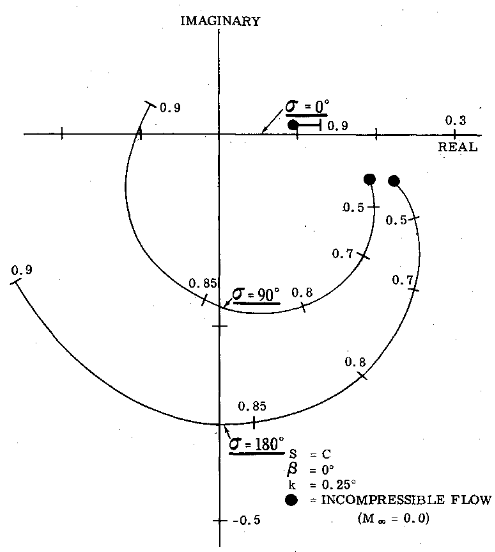

Fig. 7. Interblade-phase-angle effect on the complex fluctuating moment coefficient with the mean cascade inlet Mach number as parameter.

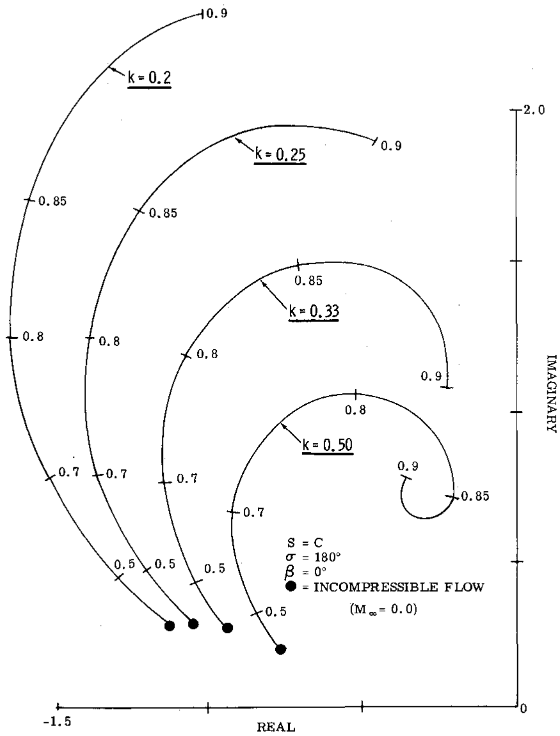

Fig. 8 Effect of reduced frequency on the complex fluctuating lift coefficient with the mean cascade inlet Mach number as parameter.

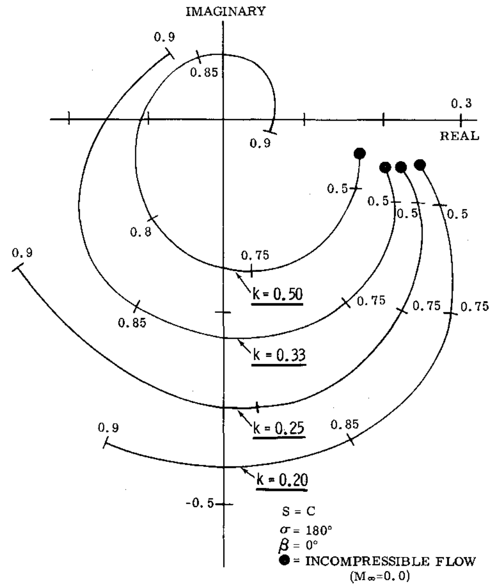

Fig. 9 Effect of reduced frequency on the complex fluctuating moment coefficient with the mean cascade inlet Mach number as parameter.

Figures 6 and 7 indicate the variations of the complex fluctuating lift and moment coefficients with interblade phase angle as the mean cascade inlet Mach number is varied from 0.0 to 0.9 with the cascade solidity equal to one, the reduced frequency equal to 0.25, and zero stagger angle. As the interblade phase angle is increased from 0° towards 180°, the absolute magnitudes of the fluctuating lift and moment coefficients increase. As the mean cascade inlet Mach number is increased, the absolute magnitudes of the fluctuating coefficients increase until they attain their maximum values at mean Mach numbers that are less than unity. Further increases in the mean Mach number beyond the peak mean Mach numbers result in the absolute magnitudes of the coefficients decreasing. The values of the peak mean Mach numbers can be seen to increase as the interblade phase angle approaches 180°. The complex phase of the fluctuating lift and moment coefficients can also be seen to change near the peak mean cascade inlet Mach numbers with the changes in phase becoming relatively large as the interblade phase angle approaches 180°. It is interesting to note that there is little change in the lift and moment coefficients with mean Mach number at an interblade phase angle equal to zero degrees. These interblade-phase-angle effects may be physically interpreted as follows. There are disturbances in the flowfield at the leading and trailing edges of the airfoils when the airfoils are out of phase with each other (the interblade phase angle is equal to 180°) due to the "pumping action" of the airfoils which is not present when the airfoils move in phase (the interblade phase angle equal to 0°). These disturbances result in the significant effects on the fluctuating lift and moment coefficients as the interblade phase angle approaches 180° previously noted.

Figures 8 and 9 show the variation with reduced frequency of the fluctuating lift and moment coefficients as the mean cascade inlet Mach number is varied over the subsonic compressible regime for a value of the cascade solidity equal to one, zero stagger angle, and a 180° interblade phase angle. The absolute magnitudes of these coefficients increase as the reduced frequency is decreased and they attain their maximum values at mean Mach numbers which are less than one. These peak Mach numbers increase in value as the reduced frequency decreases.

To demonstrate the effects of compressibility on the forces acting on a cascade of airfoils in a nonuniform subsonic flowfield as determined herein, the forced vibration response of a cantilevered airfoil cascade will now be considered.

The forced displacement of an airfoil cascade $Z_b(Y,t)$ can be expressed by Eq. (16) (Ref. 4)

$$Z_b(Y,t) = \sum_{i=1}^{\infty} \zeta_i(Y)\eta_i(t) \tag{16}$$

where Y denotes the spanwise airfoil coordinate; $\eta_i(t)$ are normal coordinates which must be determined; and $\zeta_i(Y)$ are the normalized natural mode shapes of the airfoils.

In this example, the airfoils are approximated by uniform, slender cantilevered beams of length, L, hence, the normalized natural mode shapes and frequencies are

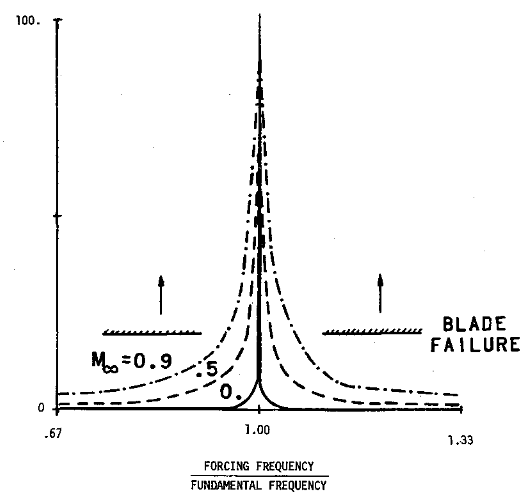

Fig. 10 Maximum forced vibration response of the airfoil tip with mean cascade inlet Mach number as parameter.

known to be the following:

$$\zeta_i(Y) = R_i[\sinh(b_iY) + \sin(b_iY)] + [\cosh(b_iY) + \cos(b_iY)] \tag{17}$$

where

$$R_i = [\cos(b_iL) - \cosh(b_iL)]/[\sinh(b_iL) - \sin(b_iL)]$$

$$b_i = (\Omega_i/a)^{1/2}; \quad a = (EI/m_b)^{1/2}$$

and Ω_i are the natural frequencies.

The response of the ith mode of motion due to a disturbing force $F_z(t)$ is given by the following differential equation for the normal coordinates:

$$M_i d^2\eta_i/dt^2 + M_i\Omega_i^2\eta_i = F_i$$

where

$$M_i = \int_0^L m_b\zeta_i^2 dY = \text{generalized mass of the } i\text{th mode} \tag{18}$$

$$F_i = \int_0^L F_z(t)\zeta_i dY = \text{generalized force of the } i\text{th mode}$$

and m_b denotes the mass per unit length of the beam.

The disturbing force directly appropriate to the analysis considered herein is the following.

$$F_z(t) = C_L \exp(j\omega t) \tag{19}$$

where C_L is the fluctuating lift coefficient determined and discussed herein and ω is the frequency of this forcing function.

For this example, the airfoil parameters chosen are representative of a first-stage high-tip speed fan blade. These parameters include a blade span of 12 in., a 4-in. chord, a thickness of 0.06 in., a cascade solidity equal to unity, a zero stagger angle, and an interblade phase angle of 90°. The reduced frequency ($k = \omega C/2U_\infty$) is dependent on the values of the forcing frequency and the mean flowfield Mach number and, therefore, the reduced frequency is not an a priori chosen constant.

Equation (18) can now be easily solved, and the forced displacement motion of the airfoil cascade due to the fluctuating lift force determined from Eq. (16). The first five natural modes and corresponding natural frequencies were used in the evaluation of the forced displacement motion.

Figure 10 shows the maximum forced vibration response of the airfoil tip over a range of forcing frequency values near the fundamental frequency of the blade with the mean inlet Mach number as a parameter. This figure indicates that the effects of compressibility are important and that the resonance peak widens with increasing values of the mean inlet Mach number.

Conclusions

This paper has considered the problem of determining the compressible fluctuating lift and moment coefficients for cascaded airfoils due to an upstream distrubance. Results have been presented which indicate the effects of cascade solidity, cascade stagger angle, interblade phase angle, and reduce frequency on these fluctuating coefficients over a mean cascade inlet Mach-number range of 0.0 to 0.9. These results clearly indicate that compressibility has a large effect upon the complex values of both the fluctuating lift and moment coefficients.

As indicated and discussed, the absolute magnitudes of the fluctuating lift and moment coefficients increase with increasing values of the cascade solidity, with an interblade phase angle approaching 180°, and with decreasing values of the reduced frequency and cascade stagger angle.

The absolute magnitudes of the fluctuating lift and moment coefficients attain maximum values at mean cascade inlet Mach numbers that are dependent upon the aeroelastic and cascade parameters, and for the values of these parameters considered herein, these peak mean Mach numbers are less than unity. These mean Mach numbers of maximum-absolute-magnitude fluctuating lift and moment coefficients increase with increasing cascade solidity, with an interblade phase angle approaching 180°, with decreasing reduced frequency and are almost independent of the cascade stagger angle. At these maximum-magnitude mean Mach numbers there is found a relatively large change in the complex phases of the fluctuating lift and moment coefficients, which appears to be analogous to an aerodynamic resonance.

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